Lecture 15 GL Theory

Boundary Conditions on $\psi(\vec{r})$
GL coherence length
Electrodynamics
GL κ-parameter

Boundary Conditions
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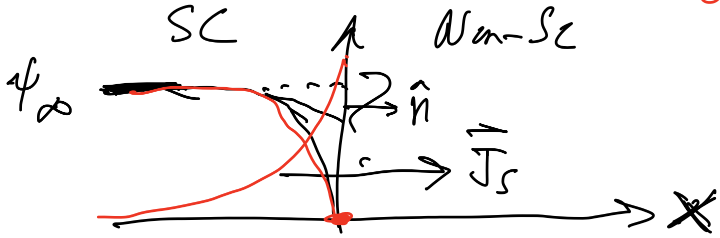


$\hat{n}\cdot\vec{J}_s = 0$

$e^* |\psi|^2 \hat{n}\cdot\vec{V}_s = 0$

$\Rightarrow |\psi|^2 \to 0$ at the interface

problem $\hat{n}\times\vec{J}_s = 0$

Not correct. Surface screening

Better B.C.

$$\hat{n}\cdot\vec{J}_s = \frac{e^*}{m^*}\,\mathrm{Re}\left\{\psi^* \hat{n}\cdot\left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)\psi\right\} = 0$$

$$\hat{n}\cdot\left(\frac{1}{i}\vec{\nabla} - \frac{e^*}{\hbar}\vec{A}\right)\psi = \frac{i}{b}\psi$$

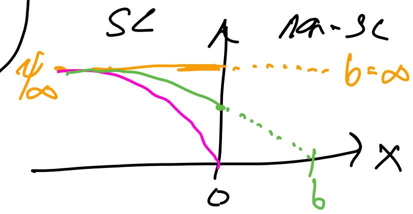
b = real number
Extrapolation length

$$\hat{n}\cdot\vec{J}_s = \frac{e^*}{m^*}\,\mathrm{Re}\left\{\underbrace{\psi^*\psi}_{n_s}\frac{i}{b}\right\} = 0$$

(Ex) $\vec{A} = 0$ 1D

$$\frac{1}{i}\frac{\partial\psi}{\partial x}\bigg|_{x=0} = \frac{i}{b}\psi(0)$$

$$\frac{\partial\psi}{\partial x}\bigg|_{x=0} = -\frac{1}{b}\psi(0)$$



$b = \infty$ Insulator
$b = 0$ Ferromagnet
b finite Normal Metal
$\Delta(\vec{r})$ in non-SC metal

GL Coherence Length

GL Equation

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)^2\psi = 0$$

$\vec{A} = 0$

Divide through $\alpha\psi_\infty$ $\quad (|\psi_\infty|)^2 = -\alpha/\beta$

$$\frac{\psi}{\psi_\infty} + \frac{\beta}{\alpha}|\psi|^2\frac{\psi}{\psi_\infty} - \frac{\hbar^2}{2m^*}\frac{1}{\alpha}\nabla^2\psi/\psi_\infty = 0$$

$-\frac{1}{|\psi_\infty|^2}$

$f \equiv \psi/\psi_\infty \qquad 0 \le |f| \le 1$

$$f - f^3 - \underbrace{\frac{\hbar^2}{2m^* \alpha}}_{\text{Dimensionless}} \nabla^2 f = 0 \qquad \{f\} = 1$$

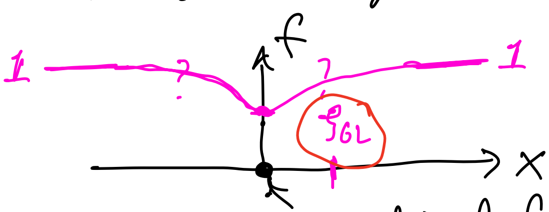
GL coherence Length

$$\xi_{GL}^2 \equiv \frac{\hbar^2}{2m^*|\alpha|} \sim \frac{1}{1-t} \qquad t = \frac{T}{T_c}$$

$$\boxed{f - f^3 + \xi_{GL}^2 \nabla^2 f = 0}$$

$$\xi_{GL} \sim \frac{1}{\sqrt{1-t}} \rightarrow \quad \text{Diverges as } T \to T_c$$

Linearize this equation.



Perturbation

$$f(x) = 1 + g(x)$$
$$g(x) \ll 1$$

$$(1+g) - (1+g)^3 + \xi_{GL}^2 \frac{d^2 g}{dx^2} = 0$$

$$\cancel{1} + g - (\cancel{1} + 3g + \cdots) + \xi_{GL}^2 \frac{d^2 g}{dx^2} = 0$$

$$-2g = \xi_{GL}^2 \frac{d^2 g}{dx^2}$$

$$\frac{d^2 g}{dx^2} = +2\frac{g}{\xi_{GL}^2} \Rightarrow \qquad g(x) = A\, e^{\pm\sqrt{2}\,x/\xi_{GL}}$$

$$f(x) = 1 - |A|\, e^{-\sqrt{2}\,x/\xi_{GL}}$$

ξ_{GL} is the characteristic length on which $|\psi|$ varies.

$\xi_{GL}(T) \to \infty$ as $T \to T_c$

$|\psi(\vec{r})|$ becomes uniform in space as $T \to T_c$

$$\boxed{\begin{aligned} \xi_{GL}(T) &= \frac{\Phi_0}{2\pi\sqrt{2}\,\mu_0 H_c(T)\,\lambda_{eff}(T)} \\ H_c(T) &= \frac{1}{2\pi\sqrt{2}\,\mu_0} \frac{\Phi_0}{\xi_{GL}(T)\,\lambda_{eff}(T)} \end{aligned}}$$

$\Phi_0 = \frac{h}{2e} = \frac{h}{e^*}$ Flux quantum

$\sqrt{\langle \rho^2 \rangle}$ RMS Radius of a Cooper Pair

$$\sqrt{\langle \rho^2 \rangle} = \frac{2}{\sqrt{3}} \frac{\hbar v_F}{W}$$

W = Binding energy / pair Cooper

BCS Coherence Length $\quad \xi_0 = \dfrac{\hbar v_F}{\pi \Delta(0)}$ $\quad T = 0$

Aluminum $\quad v_F = 2.06 \times 10^6$ m/s $\quad \Delta(0) = 340\ \mu eV$

$\xi_0 = 1.25\ \mu m$ $\quad$ Large

Niobium $\quad \xi_0 = 94$ nm

$YBa_2Cu_3O_7$ $\quad \xi_0 = 2.6$ nm $\quad$ Short

$$\frac{\xi_{GL}(T)}{\xi_0} = \underbrace{\frac{\pi}{2\sqrt{3}}}_{0.91} \frac{\lambda_L(0)\, H_c(0)}{\lambda_{eff}(T)\, H_c(T)}$$

$$\xi_0 = \frac{\sqrt{3}}{\sqrt{2}\,\pi^2} \frac{\Phi_0}{\mu_0 \lambda_L(0) H_c(0)}$$

$\lambda_{eff} \geq \lambda_L$

Electrodynamics of Superconductors

Clean Limit

Dirty Limit.

Drude Model for the normal state

$$\sigma_1(\omega) = \frac{ne^2\tau/m}{1+(\omega\tau)^2}$$

τ = momentum relaxation time

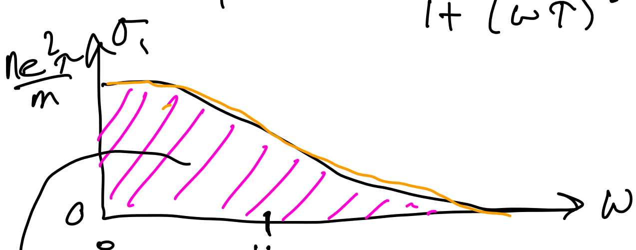
n = Total electron density

$0 < \omega < \infty$

$2\Delta(0)/\hbar$

$1/\tau$

n/m oscillator strength



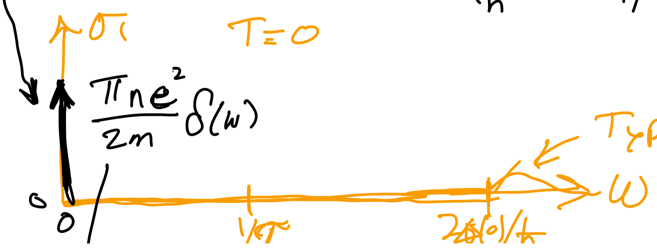
Sum Rule $\quad \displaystyle\int_0^\infty \sigma_1(\omega)\, d\omega = \frac{\pi n e^2}{2m}$ $\quad$ independent of τ

$$= \frac{\pi}{2} \frac{1}{\mu_0 \lambda_L^2}$$

Clean Limit.

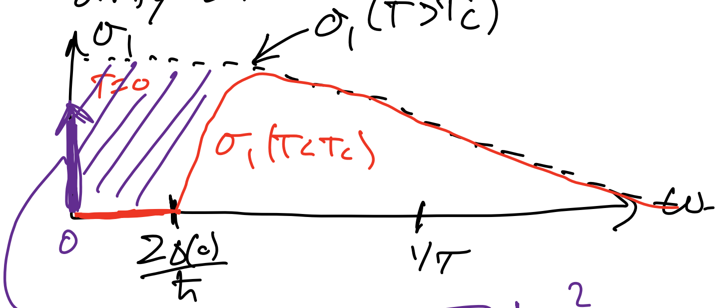
$$\frac{2\Delta(0)}{\hbar} \gg \frac{1}{\tau} \Rightarrow v_F \tau \gg \frac{\hbar v_F}{2\Delta(0)}$$

$\ell_{MFP} \gg \xi_0$ clean limit.



$\downarrow \quad \frac{\pi}{2}\frac{1}{\mu_0 \lambda_{eff}^2} \Rightarrow \lambda_{eff} \simeq \lambda_L$ in the clean limit.

Dirty Limit



$$\sigma_1(\omega=0) = \frac{\pi n' e^2}{2m}\delta(\omega)$$

$n' < n$

$$\int_0^{\Delta(0)/\hbar} \sigma_1(\omega)\, d\omega = 0$$

$$n' = n\,\frac{2}{\pi}\frac{l_{MFP}}{\xi_0} \ll n$$

$l_{MFP} \ll \xi_0$ Dirty Limit.

$$\frac{\lambda_{eff}}{\lambda_L} = \sqrt{\frac{\pi}{2}\frac{\xi_0}{l_{MFP}}} \gg 1$$

Large screening length

GL κ parameter Dimensionless Quantity

$$\kappa \equiv \frac{\lambda_{eff}(T)}{\xi_{GL}(T)} = \frac{2\pi\sqrt{2}\,\mu_0 H_c(T)\,\lambda_{eff}^2(T)}{\Phi_0}$$

$$\sim \frac{1}{1+t^2} \sim \text{constant near } T_c.$$

Exact numerical values.

$\kappa = 0.96\,\frac{\lambda_L(0)}{\xi_0}$ clean local limit $l_{MFP}\to\infty$

$\kappa = 0.715\,\frac{\lambda_L(0)}{l_{MFP}}$ dirty limit $l_{MFP} \ll \xi_0$

Al	$\lambda_L(0) = 16$ nm	$\xi_0 = 1.25\,\mu m$	$\kappa = 0.01$	Type-I
Sn	$= 25$ nm	$\xi_0 = 300$ nm	$\kappa = 0.11$	
YBCO	$\lambda_L = 150$ nm	$\xi_0 = 2.6$ nm	$\kappa = 58$	Type-II

$K = \frac{1}{\sqrt{2}}$ Dividing line between type I and type II.

$\psi = |\psi| e^{i\theta}$

$|\psi|^2 = n_s(\vec{r}) =$ # density of SC electrons.

$\underline{n} = \frac{N}{V}$ ~~n_s~~ $N_s = \frac{N}{2}$